ANALYSIS OF REINFORCED CONCRETE STRUCTURES WITH OCCURRENCE OF DISCRETE CRACKS AT ARBITRARY POSITONS

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SUMMARY

A nonlinear analysis of in-plane loaded plates is presented, which eliminates the disadvantages of the smeared crack approach. The paper discusses the elements used and the computational method. An example is shown in which one or more discrete cracks are dominant.

1. INTRODUCTION

In reinforced and prestressed concrete structures the post cracking behaviour, the collapse mechanism and the magnitude of the failure load are in most cases highly determined by the system of cracks that develops in the concrete. It is therefore not surprising that in finite element programs for the analysis of the nonlinear behaviour of concrete structures, besides the modelling of the constitutive relations, considerable attention is devoted to the inclusion in these programs of the occurrence of concrete cracks. In the literature two methods of schematizing the cracks are to be distinguished, namely: a method based on the possibility of discrete cracks along the boundaries of the elements and a method in which the cracks are assumed to be distributed over the element or over parts thereof. Each method has certain advantages and disadvantages. The aim of the reported study was to develop a model in which the advantages of both methods were combined (ref. 1). The model has been set up for the analysis of two-dimensional in-plane loaded reinforced or prestressed concrete structures. In the model are considered the various types of nonlinear material time-independent and time-dependent behaviour, the performance of the boundary layer between steel and concrete and the occurrence of discrete cracks within the structure.

The model is based on the finite element approach. For describing the structure two types of elements have been developed: a triangular thin plate element for schematizing the concrete and a bar element for describing the reinforcing steel or prestressing steel plus the bond zone with the surrounding concrete. Both these elements are based on the hybrid method with (what has been called) natural boundary displacements. It is characteristic of these elements that the stresses at their boundaries are always in equilibrium with one another and with the internal loading.

Besides taking account of the discontinuity in the displacements on each side of a crack, the model also takes account of discontinuity across a crack of the normal stresses in the direction of the crack. The method of initial strains is used for dealing with the nonlinear behaviour of the materials, the displacements at the crack and the slip of the reinforcement.

The development of this model, which has been called the MICRO-model, forms a part of the Dutch research project "Concrete Mechanics" (in Dutch: Betonmechanica). In this project concrete structures are studied along two parallel lines of basic experiments and computational methods. The subprojects for basic experiments concentrate on the fundamental behaviour of bond zones and on the phenomenon of force transfer in cracks. The results of this experimental work is fed into the subproject for computational methods. Apart of the here described model for two-dimensional in-plane loaded structures, also a model has been derived for the special case of plane framed structures in which linear elements are used allowing for normal strains, bending strains and shear strains. Because of the use of greater elements, this last model was called the MACRO-model. This paper will be restricted to the MACRO-model.

Discrete cracks versus "smeared-out" cracks

The method with discrete cracks:

- gives better insight into the relative displacements at a crack and the crack spacing;
- offers the possibility of describing the stress peaks and the dowel forces in the steel at a crack;
- can take account of the relationship between aggregate interlock and displacements at a crack;
- is often better able to schematize dominant cracks and their effect on behaviour.

A serious disadvantage of this method was that cracking was restricted to occur only along the element boundaries (ref. 2,3). This results in a high degree of schematization of the cracking pattern and considerable dependence on the subdivision into elements. Also in consequence of the detachment of the elements the system of equations must each time be re-established and inverted or decomposed.

Because of these disadvantages, in general, the discrete crack model has been abandoned in favour of the approach in which a crack is smeared or spread out over a whole element or over part of an element. The crack is thus incorporated into the stiffness properties of the concrete, which becomes anisotropic in consequence (refs. 4,5). Its great advantage is that cracking is conceived as a phenomenon like plastic deformation and can therefore be analyzed by the same methods, with which a good deal of experience has already been gained. The disadvantages of this method are due to "smearing out" the cracks. Especially the assumption about the stiffness perpendicular to the crack in an element with few or no reinforcement forms a problem. The reason is that in reality this stiffness not only depends on the element and the position of the crack herein but also on the circumstance if the element

is a link in a series connection of elements or a link in a parallel connection of elements. With this model the crack spacings and displacements at the cracks are difficult to calculate, even if a fine-meshed network of elements is used. This has its repercussions on the modelling of the aggregate interlock which highly depends on the displacements at the crack. Whether these drawbacks constitute a serious objection will depend on the kind of structure to be analysed.

In the MICRO-model a method of crack schematization is adopted which combines the advantages of both methods by treating cracks as (what they in reality are) discrete material boundaries for which the displacements and the normal stresses in the crack direction may be different on both sides. These "discrete" cracks may pass through the element mesh at any place in any direction and are continuous over the element boundaries.

Hybrid element model and natural boundary displacements

The hybrid element model with natural boundary displacements is used for the derivation of the force-deformation relations per element. In this model an assumption is made with regard to the distribution of the stresses in the element. The distribution of the displacements of the element boundaries is likewise assumed. This model offers the following advantages:

- the distribution of the stresses in the various types of element can be suitably interadjusted;
- discontinuous distribution of the displacements in an element can be taken into account quite simply in this model. Such discontinuity occurs if a crack passes through the element;
- the favourable experience previously gained with this type of finite element model can be used;
- the model offers the possibility of adding extra stress functions for describing special situations to the stress functions already existing;
- by adjusting the description of the displacements of element boundaries to the stress distribution at these boundaries it is ensured that the conditions of equilibrium are exactly satisfied at the boundaries. The advantage of this is that the stresses at a section along the element boundaries are always in equilibrium with the external loads.

The method of adjusting the description of the displacements of the element boundaries to the stress field so that inter-edge equilibrium is achieved is also called the method of natural boundary displacements (ref. 6). In this method we use for the description of the element boundary displacements a separate set of degrees of freedom per element boundary instead of the usually employed degrees of freedom in the element corners. In the next chapter the characteristics of the developed elements will be briefly discussed. For an extensive derivation see reference 1.

2. USED ELEMENT

Triangular plate element

The concrete is described by triangular thin plate elements. For the uncracked element we use per element boundary four degrees of freedom for the description of a linear displacement distribution in normal and tangential direction. In the element linear interpolation functions are used for the description of the stresses.To restrict in an element with twelve degrees of freedom the number of stressless displacement possibilities to the three rigid-body displacement modes, it is necessary to have at least nine independent stress parameters. A linearly distributed stress field for a thin plate which satisfies the internal equilibrium conditions in every point of the element only has seven independent stress parameters. So to satisfy the condition of nine independent stress parameters the second equation of Cauchy which states that σ_{xy} equals σ_{yx} in every point of the plate, is relaxed into the condition that the area integral per element of the shear stresses σ_{xy} and σ_{yx} must be equal ($\int_{v_e} \sigma_{xy}\, dv = \int_{v_e} \sigma_{yx}\, dv$).

Because the linear stress distribution per element boundary is uniquely represented by the four stress resultants per element boundary,full inter-edge equilibrium is achieved.

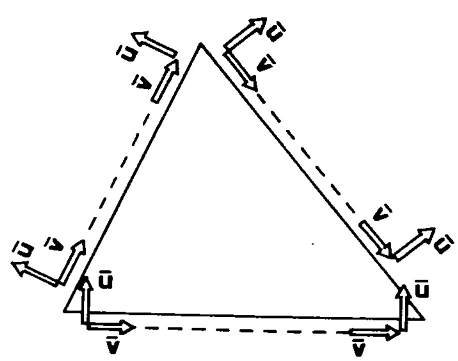

Figure 2.1 Degrees of freedom of an uncracked plate element.

If a crack has to occur in an element, this crack is assumed to form, in a straight line from one boundary of the element to another. Within a crack three additional degrees of freedom are introduced, two for the description of a linear varying crack opening (u_i , u_j) and one for the description of the parallel shift (v). (See figs. 2.1 to 2.4.)

In the vicinity of a crack the stresses may vary greatly due to dowel forces in the rebars or bond stresses between rebars and concrete.
To take account of these stress variations and the possibility of a discontinuity at a crack of the normal stress in the crack direction , the linear stress field of the uncracked element is extended for a cracked element with a stress field which is discontinuous across the crack.

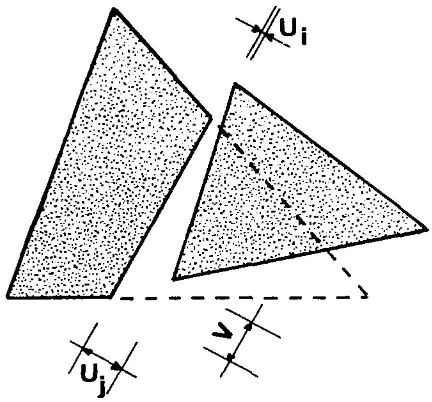

Figure 2.2 Displacement possibilities at a crack.

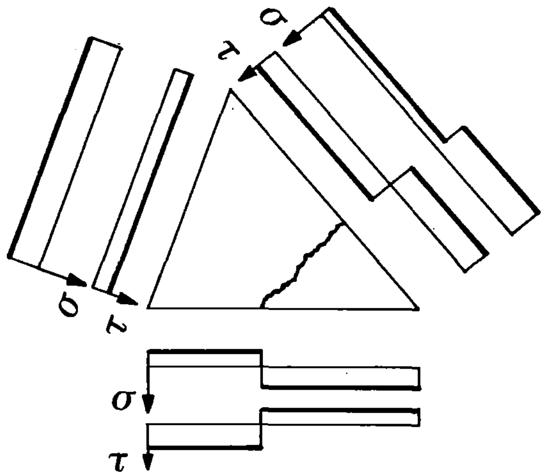

Figure 2.3 Distribution of the stresses, from the additional stress field, along the boundaries of a plate element with one crack.

To preserve full inter-edge equilibrium in a crack-crossed element boundary it is necessary to add to the linear displacement interpolation a discontinuous displacement interpolation. This is done per crack-crossed element boundary with the additional degrees of freedom Δu^0 and Δv^0.

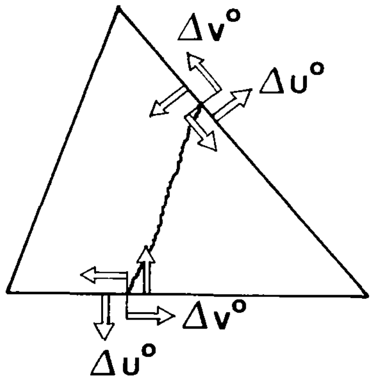

Figure 2.4 Extra degrees of freedom at cracked element boundaries.

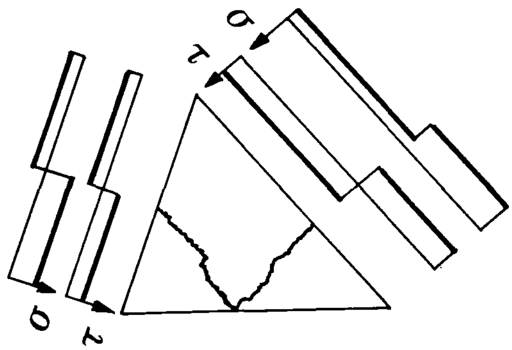

Figure 2.5 Distribution of the stresses, from the additional stress field, along the boundaries of a plate element with two cracks.

In an element a second crack is permitted only if this crack runs from the uncracked edge to the intersection of the first crack and the element boundary (see figure 2.5). Now the additional stress field is discontinuous over both cracks and we find additional degrees of freedom along all three element boundaries.

Rebar element

A linear bar element is used for the schematization of the embedded steel and the properties of the contact zone for bond between steel and concrete. The distribution of the forces in the bar element is adjusted for the distribution of the stresses along the boundaries of the triangular plate element with which these bar elements are to be associated. In the uncracked plate element a linear stress distribution has been adopted . So the shear stress τ along the rebar and the normal stress σ are also linear, which corresponds to a quadratic distribution for the normal force *(F)* and shear force *(S)* in the bar element. The distribution of the shear force in an element has been so chosen that the average shear force is always zero. This ensures that the bending moments in the bar remain small and that they are zero at the ends of the bar. It is still a point of discussion if this is permitted for all situations.

This assumption was necessary to restrict the number of stressless displacement modes to three. Because of the small influence of the shear flexibility on the structural behaviour it seems to be an allowable assumption.

$$\begin{bmatrix} F \\ \tau \\ S \\ \sigma \end{bmatrix} = \begin{bmatrix} 1 & s & s^2 & 0 & 0 \\ 0 & 1 & 2s & 0 & 0 \\ 0 & 0 & 0 & l^2-3s^2 & 2sl-3s^2 \\ 0 & 0 & 0 & -6s & 2l-6s \end{bmatrix} \begin{bmatrix} \beta(1) \\ \\ \beta(5) \end{bmatrix}$$

Stress interpolation for uncracked rebar element; s is the coordinate along the element edge and l the length of the edge

In expectation of the results of the other study on the real properties of the boundary layer the constitutive equations for the combined steel/boundary layer element are taken as

$$\begin{bmatrix} \varepsilon \\ \Delta_{||} \\ \gamma \\ \Delta_{\perp} \end{bmatrix} = \begin{bmatrix} \frac{1}{AE} & 0 & 0 & 0 \\ 0 & \frac{1}{K} & 0 & 0 \\ 0 & 0 & \frac{1}{D} & 0 \\ 0 & 0 & 0 & \frac{1}{B} \end{bmatrix} \begin{bmatrix} F \\ \tau \ (\frac{dF}{ds}) \\ S \\ \sigma \ (\frac{dS}{ds}) \end{bmatrix}$$

where: ε = strain of the steel
$\Delta_{||}$ = slip in boundary layer
γ = deformation in steel due to shear force
$\Delta_{\perp}$ = indentation of boundary layer
A = cross-sectional area of steel
E = modules of elasticity of steel
K = elastic stiffness of boundary layer with respect to slip
D = dowel rigidity of steel
B = elastic stiffness of boundary layer with respect to indentation

The displacements at the element boundaries are described with the aid of the displacement quantities $\overline{u}$ and $\overline{v}$ along the outside of the boundary layer and the displacements u and v at the outer ends of the steel bar.

The first set of degrees of freedom corresponds with the degrees of freedom of the plate element while the second set provides for the continuity of the normal and shear force over the length of the reinforcement.

If the bar element is intersected by a crack, then -as in the plate element- the stress functions and displacement function are extended by adding extra fields. These fields are compatible with the extra stresses and displacements used in the plate element.

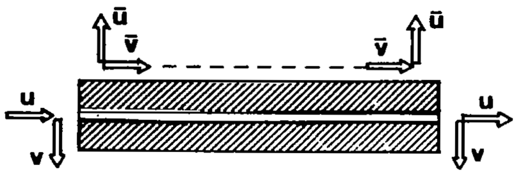

Figure 2.6 Degrees of freedom of an uncracked bar element.

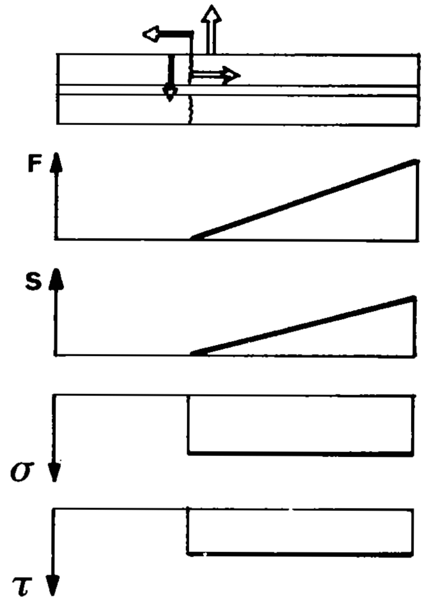

Figure 2.7 Extra stress fields and degrees of freedom in a bar element intersected by a crack.

3. COMPUTATIONAL METHOD

The finite element stiffness relations and equations were derived by using a Galerkin approach for the kinematic relations and the equilibrium conditions (see reference 1). This results for a structure without any cracks in the next set of equations:

$$S\underline{v}^0 = \underline{k} + A\underline{\varepsilon}^I - B\underline{q}$$

Herein v^0 are the degrees of freedom, k represents the applied load, ε^I is the sum of all initial strains and q is the volume load. In the method of analysis envisaged in the MICRO-model, the strains are split up into an elastic part and an initial part. The elastic strains are those which would occur if the material displayed ideal linearly elastic behaviour. The initial strains are used to account for all nonlinearities such as plastic deformation, creep, shrinkage, etc. When a load increment is applied the set of equations is solved iteratively by adjusting the initial strains until all criteria of nonlinearity are fulfilled to a certain accuracy.

When cracks occur, an additional stress field with stress parameters $\overline{\beta}$ and additional degrees of freedom $\overline{v}^0$ are introduced (v.s.).
Now it can be shown that the set of equations is replaced by two systems of equations:

$$S\underline{v}^0 = \underline{k} + A\underline{\varepsilon}^I - B\underline{q} - C\underline{v}^{cr} - D\underline{\overline{\beta}}$$

$$\overline{S}\underline{\overline{v}}^0 = \overline{A}\underline{\varepsilon}^I - \overline{C}\underline{v}^{cr} - E\underline{v}^0$$

Herein v^{cr} are the number of degrees of freedom on the crack faces (vs.) and v^0 the additional degrees of freedom on the element boundaries at the point of intersection with a crack.
The split up of the equations in two sets is done to avoid the alteration of the origional system matrix S and the renumbering of the degrees of freedom v^0. During the iterative solution procedure both sets of equations are solved in sequence. In this procedure we calculate $\overline{v}^0$ from the first set of equations using for the additional stress parameters β the value from the preceding iteration. In each iteration the initial strains ε^I and the internal crack displacements v^{cr} are adjusted to the criteria of nonlinearity c.q. the stress conditions for a crack.
To take into account the internal stress redistribution due to a crack, one element crack at a time is allowed to occur. Only when the normal stresses on the crack surfaces have become sufficiently low, can another cracked element occur.
Each time when a new crack is formed the matrix $\overline{S}$ has to be formed and decomposed again.
Because the bandwidth of this matrix stays very small this requires much less time than a reformation and decompositon of matrix S would take.

To decide when an element is cracked and to determine the direction of the crack we use the average stresses over an element. When these stresses are in the range in which the crack criterion is valid and supercedes the criterion more than will occur in other elements, a crack is assumed to form (with the restriction that the normal stresses on the existing crack faces are small enough). A crack is placed through the centre of the triangular element, except if a crack already ends on the boundary of the element. In that case the new crack proceeds from this existing crack.
In reality there is a local stress peak near the tip of a crack. This causes further spreading of a crack, even if the average stresses in the vicinity thereof -apart from the stress peaks- are below the cracking criterion. In the MICRO-model these highly localized stress fields are not included. The effect that, in an element adjacent to the end of an existing crack, a crack will develop at lower average stresses than it would if there were no cracks present, is here dealt with by reducing the cracking criterion for these elements. The calculations that have been performed show a reduction to about 0.7 to be satisfactory. A crack, once it has been introduced into the model, remains in existence. The procedure does however take account of the possibility that, on further loading or unloading the structure, it may occur that a crack closes up again by compression, but as soon as tensile stresses act across a closed crack, the latter opens again. Transfer of compressive stresses across a crack is possible only for zero crack width.

For determining the displacements in the cracks and the initial strain for the elasto-plastic materials a fictitious visco-plastic model is used. By doing this the iteration process can be conceived as a fictitious creep process with a time interval Δt between each two successive iterations and a loading of the viscous element equal to the unbalanced stresses $(\Delta\sigma)$. Per iteration the increase in the internal crack displacements or initial strains is

$$\Delta v^{CR} \; (\Delta\varepsilon^{I}) = K\Delta\sigma$$

To ensure that the iteration process is stable the value of K must not be taken too large (see reference 7). The number of iterations needed per load increment is highly influenced by the number of cracks present in the structure. In order to speed up the iteration process, whenever a number of cracks have formed, the system matrices S and $\overline{S}$ are changed in order to take into account the condition that the normal stresses on the faces of open cracks must become zero.

4. EXAMPLE OF REINFORCED BEAM WHICH FAILS IN SHEAR

After several calculations in which the MICRO-model had proven its ability to simulate bending failure (see reference 1), a reinforced beam which fails in shear was analysed with the model. This beam is one of a series of beams which were tested in the Stevin Laboratory of the Delft University of Technology in the Netherlands in a program of research to investigate the influence of beam depth and crack roughness on the shear failure load (ref. 8). The beam was loaded as shown in figure 4.1.

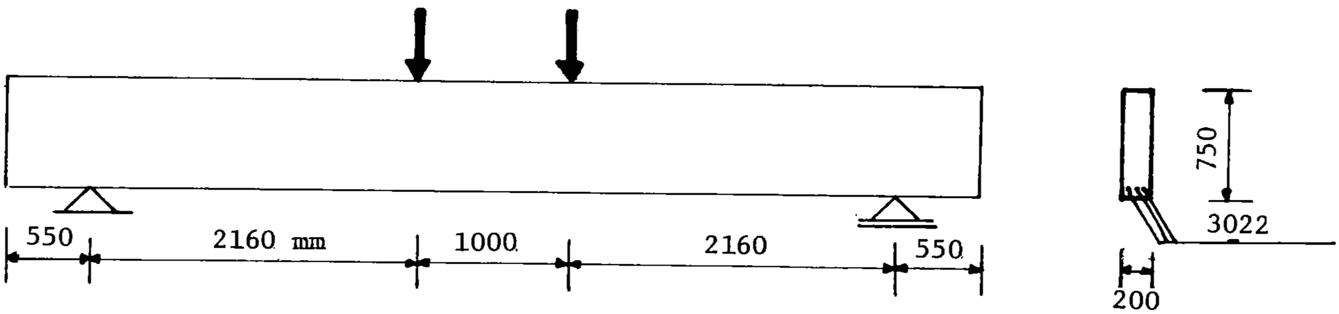

Figure 4.1 Shape and manner of loading of tested beam.

On account of symmetry of the structure, the boundary conditions and the loading, it was sufficient to confine the analysis to one half of the structure. The network of elements, the restraints and support and the external loading of this half structure have been shown in figure 4.2.

In the experiment as well as in the analysis abrupt failure occured, caused by crushing of the concrete at the tip of an inclined (shear) crack. At failure the stresses in the rebars were still below the yield stress.

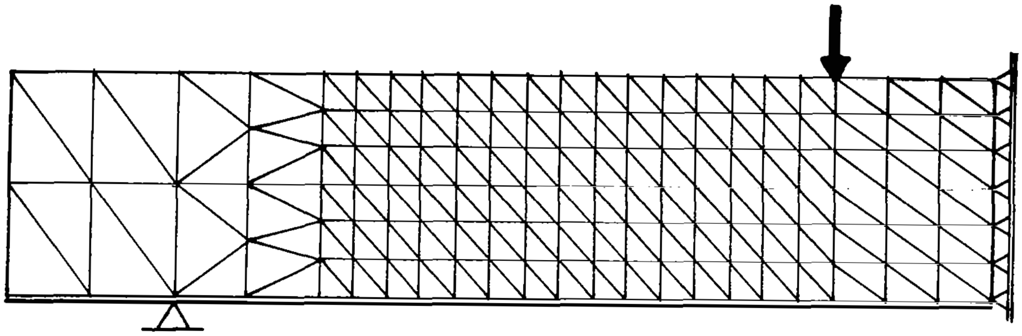

Figure 4.2 Network of element.

The experimentally determined failure load and the failure load found from the analysis were very close to each other (112,1kN and 112,4kN). The load deflection curves for the experiment and the analysis are given in Fig. 4.3.

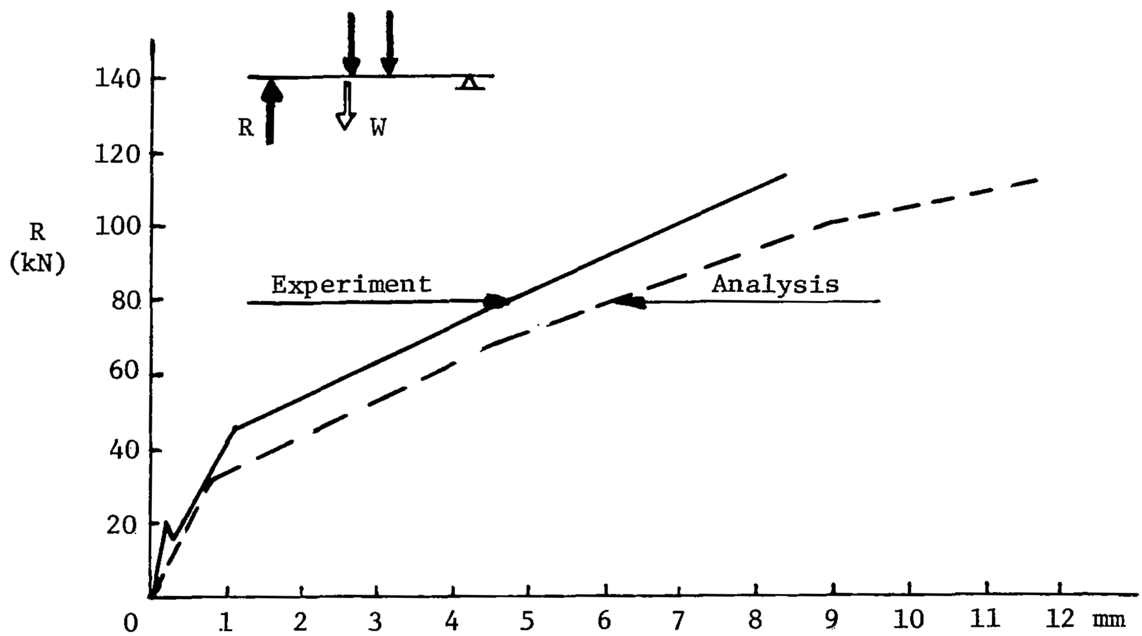

Figure 4.3 Load-deflection diagram.

It follows from the load-deflection curves that the analysis leads to a somewhat lower stiffness than was registered in the experiment. An explanation for this lower stiffness may be a too low tensile strength for the concrete in the analysis, which results in the premature occurrence of cracks and a bend in the load-deflection curve at a lower value of the load than in the test. The maximum bond stress between steel and concrete may have been chosen too low as well.

Fig. 4.4 shows the crack patterns, just before failure, according to experiment and analysis. For convenience of comparison the reflection of the right part of the beam has been displayed in this figure.

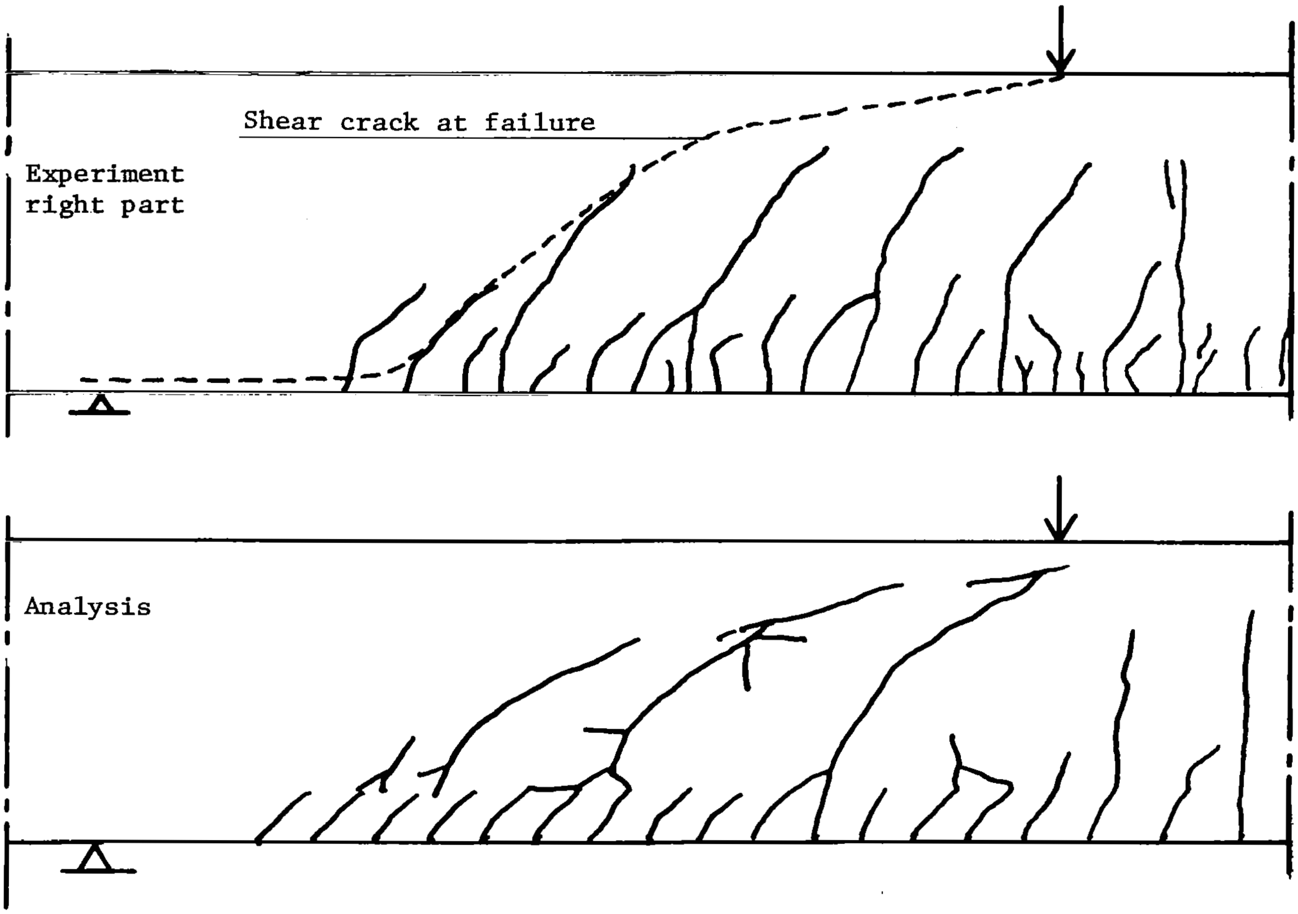

Figure 4.4 Crack pattern experiment and analysis.

It can be seen from these crack patterns that the beam fails due to an inclined shear crack in the experiment. This inclined crack was also found in the analysis. In fact there is a very good agreement between the crack patterns of the test and the analysis. Although not much information about the width of the cracks in the experiment is available, it seems that in the analysis somewhat larger crackwidths were found than in reality. This corresponds with the smaller stiffness as discussed above and can also be the result of a slightly low value for tensile strength of concrete and maximum bond stress.

5. SUMMARY AND CONCLUSIONS

A finite element program has been presented for the analysis of two-dimensional in-plane loaded concrete structures. The program makes use of seperate elements for the description of the concrete and the rebars including the bond zone with the surrounding concrete. When cracks occur they are handled as being discrete. Displacements and stresses may be discontinuous across a crack. Cracks may pass through the finite element mesh at any place in any direction and are continuous over the element boundaries.

The elements are based on the hybrid method with natural boundary displacements, resulting in stresses at the inter-element boundaries which are always in equilibrium with one another and with the external loading. The model takes care of the different types of nonlinear material behaviour. Comparison of the results of experiments with the results of analyses shows that the model is capable of obtaining a good prediction of the deformation, crack pattern, crack widths, failure load and internal stress distribution of concrete structures under in-plane static loading.

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